# 10. Rovnice a nerovnice s absolutními hodnotami

DEF. ABSOLUTNÍ HODNOTA REÁLNÉHO ČÍSLA $\underline{a}$ $(a \in R)$ je definována takto:

1. pro $a \geqq 0$ je $|a| = a$
2. pro $a < 0$ je $|a| = -a$ (číslo opačné)

VĚTY: Pro $\forall a \in R$:

$|a| \geqq 0$ $\qquad |a| = |-a| \qquad |a \cdot b| = |a| \cdot |b| \qquad \left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ $(b \neq 0)$

Př. $\underbrace{|3| = 3 \quad |-3| = (-3) = 3}_{|3| = |-3|}$

$|-x| = |(-1) \cdot x| = |-1| \cdot |x| = 1 \cdot |x| = |x|$

$|-2x| = |-2||x| = 2|x| = |2x|$

$\underbrace{|2x| = |2| \cdot |x| = 2|x|}_{\text{lze přímo}}$

$\underbrace{\left|\frac{2}{x}\right| = \frac{|2|}{|x|} = \frac{2}{|x|}}_{\text{přímo}}$ $(x \neq 0)$

GEOMETRICKÝ VÝZNAM

Číslo $|a|$ je ROVNO VZDÁLENOSTI OBRAZU ČÍSLA $\underline{a}$ na číselné ose OD POČÁTKU (od obrazu čísla 0)

Číslo $|a-b| = |b-a|$ je rovno VZDÁLENOSTI OBRAZŮ ČÍSEL $a, b$ na číselné ose

$|a| \quad |a|$

$-a \quad 0 \quad a$

$|a-b| = |b-a|$

$a \quad b$

## Příklady

① Řeš v R NA ZÁKLADĚ GEOM. VÝZNAMU – TYP NEZNÁMÁ V ABS. H., KOEFICIENT U NEZN. 1

a) $|x| = 6$
$[x_0 = 0]$

$-6 \quad 0 \quad 6$

$\mathcal{K} = \{-6, 6\}$

$|x| = 0$
$[x_0 = 0]$

$0$

$\mathcal{K} = \{0\}$

$|x| = -2$
$[x_0 = 0]$
$[\forall x \in R: |x| \geqq 0]$

$\mathcal{K} = \emptyset$

[nd. nemůže být -2]

$|-x| = 2$
$|x| = 2$
$[x_0 = 0]$

$-2 \quad 0 \quad 2$

$\mathcal{K} = \{-2, 2\}$

HLEDÁME BODY, KTERÉ MAJÍ OD NULOVÉHO BODU POŽADOVANOU VZDÁL.

b) $|x+3| = 4$
$\left[\begin{matrix} x+3=0 \\ x_0=-3 \end{matrix}\right]$

$-7 \quad -3 \quad 1$
$\qquad x_0$

$\mathcal{K} = \{-7, 1\}$

$|4-x| = 2$
$|x-4| = 2$
$\left[\begin{matrix} x-4=0 \\ x_0=4 \end{matrix}\right]$

$2 \quad 4 \quad 6$

$\mathcal{K} = \{2, 6\}$

$|-x-4| = 2$
$|(-1)(x+4)| = 2$
$|-1| \cdot |x+4| = 2$
$|x+4| = 2$
$\left[\begin{matrix} x+4=0 \\ x_0=-4 \end{matrix}\right]$

$-6 \quad -4 \quad -2$

$\mathcal{K} = \{-6, -2\}$

$|x-3| = -2$
$[\forall a \in R: |a| \geqq 0]$

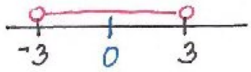
$\mathcal{K} = \emptyset$

[nd. bod $\underline{x}$ a 3 nemůže být -2]

ZOPAKOVAT ČL. 8 (Absolutní hodnota)
M1 – ČÍSELNÉ OBORY

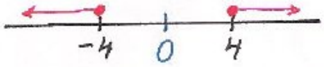
② Řešte v R

a) $|x| < 3$ $[x_0 = 0]$



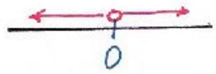
$\mathcal{K} = (-3, 3)$

$|x| \geqq 4$ $[x_0 = 0]$



$\mathcal{K} = (-\infty, -4\rangle \cup \langle 4, +\infty)$

$|x| > 0$ $[x_0 = 0]$



$\mathcal{K} = R - \{0\}$

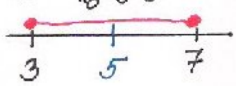
$[\mathcal{K} = (-\infty, 0) \cup (0, +\infty)]$

$|x| < 0$ $[x_0 = 0]$

$[\forall x \in R : |x| \geqq 0]$

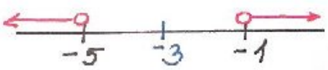
$\mathcal{K} = \emptyset$

b) $|x-5| \leqq 2$ $\left[\begin{matrix} x-5=0 \\ x_0 = 5 \end{matrix}\right]$



$\mathcal{K} = \langle 3, 7 \rangle$

$|x+3| > 2$ $[x_0 = -3]$



$\mathcal{K} = (-\infty, -5) \cup (-1, +\infty)$

$|x+1| \geqq 0$ $[x_0 = -1]$

$\mathcal{K} = R$

$|x-2| < 0$ $[x_0 = 2]$

$[\forall a \in R : |a| \geqq 0]$
$a = x-2$, $\Rightarrow$
$|x-2| < 0$ nemá řeš.

$\mathcal{K} = \emptyset$

! KOEFICIENTY U NEZNÁMÉ MUSÍ BÝT 1 -> viz násl. příklad
ROZLIŠUJ ŘEŠENÍ ROVNIC A NEROVNIC

③ Řešte v R

a) $|2x-3| = 1$ $|:2$

$|x - \frac{3}{2}| = \frac{1}{2}$

$\left[\begin{matrix} x - \frac{3}{2} = 0 \\ x_0 = \frac{3}{2} \end{matrix}\right]$

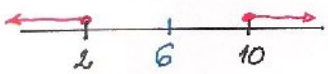
$(\frac{3}{2} - \frac{1}{2} = \frac{2}{2} = 1)$ $(\frac{3}{2} + \frac{1}{2} = \frac{4}{2} = 2)$

$\mathcal{K} = \{1, 2\}$

b) $|\frac{x}{2} - 3| \geqq 2$ $|\cdot 2$

$|x-6| \geqq 4$

$\left[\begin{matrix} x-6=0 \\ x_0 = 6 \end{matrix}\right]$

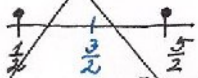


$\mathcal{K} = (-\infty, 2\rangle \cup \langle 10, +\infty)$

NELZE !

$|2x-3| = 1$

$\left[\begin{matrix} 2x-3=0 \\ 2x=3 \\ x_0 = \frac{3}{2} \end{matrix}\right]$



$[\frac{3}{2} + 1]$

c) $|-2x-1| < 3$

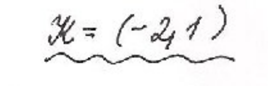
nejprve u x kladné číslo

$|(-1)(2x+1)| < 3$

$|2x+1| < 3$ $|:2$

$|x + \frac{1}{2}| < \frac{3}{2}$

$[x_0 = -\frac{1}{2}]$



$\mathcal{K} = (-2, 1)$

NEBO $|-2(x + \frac{1}{2})| < 3$

$|-2| |x + \frac{1}{2}| < 3$

$2 |x + \frac{1}{2}| < 3$ $|:2$

$|x + \frac{1}{2}| < \frac{3}{2}$

NELZE

$|-2x-1| < 3$ $|:(-2)$

$|x + \frac{1}{2}| > -\frac{3}{2}$

podobně

! POKUD SI NEJSTE JISTÍ ÚPRAVOU
=> VYUŽIJTE JINÝ ZPŮSOB ŘEŠENÍ !
(sice pomalejší, ale správný)
- viz 3a) -> 4a), 7a)

④ ŘEŠ V $R$ — VYUŽITÍM DEF. ABS. HODNOTY – VHODNÉ PRO SLOŽITĚJŠÍ S JEDNOU ABS. HODNOTOU

a) $|\overset{a}{(2x-3)}| = 1$ $\quad O = R$, $D = R$

1. $\overset{a \geq 0}{\boxed{2x-3 \geq 0}} \ldots \overset{|a| = a}{|2x-3| = 2x-3} \ldots$ rce má pak tvar $2x-3 = 1$

$2x \geq 3 \qquad\qquad 2x = 4$

$x \geq \frac{3}{2} \qquad\qquad x = 2 \in D_1$

$D_1 = \langle \frac{3}{2}, +\infty)$

$K_1 = \{2\}$

2. $\overset{a < 0}{\boxed{2x-3 < 0}} \ldots \overset{|a| = -a \text{ (výraz op.)}}{|2x-3| = -(2x-3)} \ldots$ rce má pak tvar $3-2x = 1$

$2x < 3 \qquad = -2x+3 \qquad -2x = -2$

$x < \frac{3}{2} \qquad = 3-2x \qquad x = 1 \in D_2$

výraz opačný (ke přímo)

$D_2 = (-\infty, \frac{3}{2})$

$K_2 = \{1\}$

$[\, K = K_1 \cup K_2 = \{2\} \cup \{1\} \,]$

$K = \{1, 2\}$

POROVNEJ S METODOU NUL. BODŮ – viz př. ⑦a
S ŘEŠ. NA ZÁKLADĚ GEOM. VÝZNAMU ③a)

b) $7-4x = |4x-7|$ $\quad O = R$, $D = R$

1. $4x-7 \geq 0 \ldots 7-4x = 4x-7$

$4x \geq 7 \qquad -8x = -14$

$x \geq \frac{7}{4} \qquad x = \frac{14}{8} = \frac{7}{4} \in D_1$

$D_1 = \langle \frac{7}{4}, +\infty) \qquad K_1 = \{\frac{7}{4}\}$

2. $4x-7 < 0 \ldots 7-4x = 7-4x$

$4x < 7 \qquad 0x = 0$

$x < \frac{7}{4} \qquad (0 = 0$ pl.$)$

$D_2 = (-\infty, \frac{7}{4}) \qquad K_2 = D_2 = (-\infty, \frac{7}{4})$

$K = K_1 \cup K_2 = \{\frac{7}{4}\} \cup (-\infty, \frac{7}{4})$

$K = (-\infty, \frac{7}{4}\rangle$

NUTNÉ PERFEKTNÍ ZÁPISY!

c) $u - |-2u| = 1$ $\quad O = R$, $D = R$ $\quad$ DÚ

$u - |(-2)u| = 1$

$u - 2|u| = 1$

1. $u \geq 0 \ldots u - 2u = 1$

$D_1 = \langle 0, +\infty) \qquad -u = 1$

$u = -1 \notin D_1$

$K_1 = \emptyset$

2. $u < 0 \ldots u - 2(-u) = 1$

$D_2 = (-\infty, 0) \qquad u + 2u = 1$

$3u = 1 \quad |:3$

$u = \frac{1}{3} \notin D_2$

$K_2 = \emptyset$

$K = K_1 \cup K_2 = \emptyset$

d) $|3-x| = \frac{1}{2}x + 1$ $\quad O = R$, $D = R$ $\quad$ DÚ

1. $3-x \geq 0 \ldots 3-x = \frac{1}{2}x + 1 \quad |\cdot 2$

$-x \geq -3 \quad |\cdot(-1) \qquad 6-2x = x+2 \quad |-6$

$x \leq 3 \qquad -3x = -4$

$D_1 = (-\infty, 3\rangle \qquad x = \frac{4}{3} \in D_1$

$K_1 = \{\frac{4}{3}\}$

2. $3-x < 0 \ldots x-3 = \frac{1}{2}x + 1 \quad |\cdot 2$

$-x < -3 \qquad 2x-6 = x+2$

$x > 3 \qquad x = 8 \in D_2$

$D_2 = (3, +\infty)$

$K_2 = \{8\}$

$K = K_1 \cup K_2 = \{\frac{4}{3}, 8\}$

⑤ Řešte v $\mathbb{R}$

a) $|2x-1| \leq 2$ $\qquad \mathcal{O}=\mathbb{R},\ \mathcal{D}=\mathbb{R}$

$a \geq 0 \dots |a| = a$ nemění znaménko

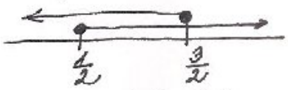
1. $2x-1 \geq 0 \quad \dots \quad 2x-1 \leq 2$

$2x \geq 1 \qquad 2x \leq 3$

$x \geq \frac{1}{2} \qquad x \leq \frac{3}{2}$

$\mathcal{D}_1 = \langle \frac{1}{2}, +\infty)$ $\qquad$ [! x musí být ale z $\mathcal{D}$!]

průnik



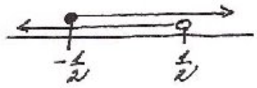
$\mathcal{K}_1 = \langle \frac{1}{2}, \frac{3}{2} \rangle$

2. $2x-1 < 0 \quad \dots \quad 1-2x \leq 2$

$2x < 1 \qquad -2x \leq 1 \quad |:(-2)$

$x < \frac{1}{2} \qquad x \geq -\frac{1}{2}$

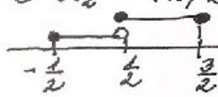
$\mathcal{D}_2 = (-\infty, \frac{1}{2})$ průnik



$\mathcal{K}_2 = \langle -\frac{1}{2}, \frac{1}{2})$

sjednocení

$\mathcal{K} = \mathcal{K}_1 \cup \mathcal{K}_2 = \langle \frac{1}{2}, \frac{3}{2} \rangle \cup \langle -\frac{1}{2}, \frac{1}{2})$



$\mathcal{K} = \langle -\frac{1}{2}, \frac{3}{2} \rangle$

b) $|x+4| - 2 \geq 4(x-3)$ $\qquad [\mathcal{O}=\mathbb{R},\ \mathcal{D}=\mathbb{R}]$

$|x+4| \geq 4x - 12 + 2$ umíme-li, nejprve zjednodušit

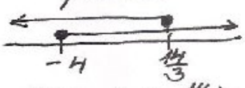
$|x+4| \geq 4x - 10$

1. $x+4 \geq 0 \quad \dots \quad x+4 \geq 4x-10$

$x \geq -4 \qquad 14 \geq 3x$

$\mathcal{D}_1 = \langle -4, +\infty) \qquad x \leq \frac{14}{3}$

průnik



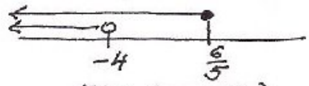
$\mathcal{K}_1 = \langle -4, \frac{14}{3} \rangle$

2. $x+4 < 0 \quad \dots \quad -x-4 \geq 4x-10$

$x < -4 \qquad 6 \geq 5x \quad |:5$

$\mathcal{D}_2 = (-\infty, -4) \qquad x \leq \frac{6}{5}$

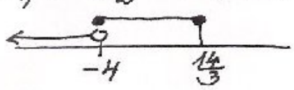
průnik



$\mathcal{K}_2 = (-\infty, -4)$

sjednocení

$\mathcal{K} = \mathcal{K}_1 \cup \mathcal{K}_2 = \langle -4, \frac{14}{3} \rangle \cup (-\infty, -4)$



$\mathcal{K} = (-\infty, \frac{14}{3} \rangle$

c) $3\left|\frac{1}{2} - 3y\right| < \frac{5}{2}$

DÚ

NEBO NEJPRVE UPRAVIT $\qquad \left|\frac{3}{2} - 9y\right| < \frac{5}{2} \quad |\cdot 2$

$|3 - 18y| < 5$ a tak dál

1. $\frac{1}{2} - 3y \geq 0 \quad |\cdot 2$ $\quad$ ($a \geq 0$, tak $a$ nemění znaménko) $\quad \dots \quad 3(\frac{1}{2} - 3y) < \frac{5}{2}$

$1 - 6y \geq 0 \qquad \frac{3}{2} - 9y < \frac{5}{2} \quad |\cdot 2$

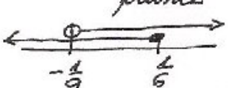
$1 \geq 6y \quad |:6 \qquad 3 - 18y < 5$

$y \leq \frac{1}{6} \qquad -18y < 2$

$\mathcal{D}_1 = (-\infty, \frac{1}{6} \rangle \qquad y > -\frac{2}{18}$

$y > -\frac{1}{9}$

průnik



$\mathcal{K}_1 = (-\frac{1}{9}, \frac{1}{6} \rangle$

2. $\frac{1}{2} - 3y < 0 \quad \dots \quad 3(3y - \frac{1}{2}) < \frac{5}{2}$

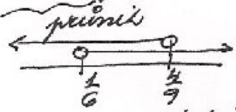
$1 - 6y < 0 \qquad 9y - \frac{3}{2} < \frac{5}{2} \quad |+\frac{3}{2}$

$1 < 6y \qquad 9y < 4 \quad |:9$

$y > \frac{1}{6} \qquad y < \frac{4}{9}$

$\mathcal{D}_2 = (\frac{1}{6}, +\infty)$

průnik



$\mathcal{K}_2 = (\frac{1}{6}, \frac{4}{9})$

sjednocení

$\mathcal{K} = (-\frac{1}{9}, \frac{1}{6} \rangle \cup (\frac{1}{6}, \frac{4}{9})$

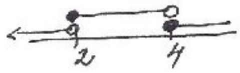
$\mathcal{K} = (-\frac{1}{9}, \frac{4}{9})$

⑥ Řešte v R

$a)\ |x-2|+|2x-8|=5$    $O=R$, $D=R$

**3. způsob: METODA NULOVÝCH BODŮ** – vhodná pro více absol. hodnot (lze ale i pro 1 abs. h.)

[ nul. b. $x-2=0$    $2x-8=0$
$x_0=2$    $2x=8$
$x_0=4$ ]



| | $D_1$ $(-\infty,2)$ | $D_2$ $\langle 2,4)$ | $D_3$ $\langle 4,+\infty)$ | |
|---|---|---|---|---|
| $\|x-2\|$ | ⊖ $2-x$ | 0 ⊕ $x-2$ | ⊕ $x-2$ | nr. 5: $x-2=3>0$ |
| $\|2x-8\|$ | ⊖ $8-2x$ | ⊖ $8-2x$ | 0 ⊕ $2x-8$ | nr. 5: $2x-8=10-8>0$ |

[1. NUL. BODY rozdělíme def. obor D na intervaly (dílčí def. obory) tak, aby jejich průnik BYLA prázdná mn. [nul. b. – mezi ]

3. v dílčích def. oborech řešíme rci (nerovnici), místo výrazu s abs. h. opíšeme výraz bez abs. h. z tabulky (v příslušném intervalu)]

[2. sestavíme tabulku pro odstranění abs. hodnoty
– nejprve zjistíme, ve kterých intervalech výraz uvnitř abs. h. nabývá kladných ⊕, resp. záporných hodnot ⊖ [postup jako u nerovnic v podíl. tvaru]
– odstraníme abs. h.: ⊕ stejný výraz
⊖ opačný výraz]

1\. $x\in(-\infty,2)\ \ldots\ 2-x+8-2x=5$
$D_1=(-\infty,2)$
$10-3x=5$
$-3x=-5$
$x=\frac{5}{3}\in D_1$
$\mathcal{K}_1=\{\frac{5}{3}\}$

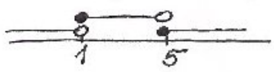
2\. $x\in\langle 2,4)\ \ldots\ x-2+8-2x=5$
$D_2=\langle 2,4)$
$-x+6=5$
$-x=-1$
$x=1\notin D_2$
$\mathcal{K}_2=\emptyset$

3\. $x\in\langle 4,+\infty)\ \ldots\ x-2+2x-8=5$
$D_3=\langle 4,+\infty)$
$3x-10=5$
$3x=15$
$x=5\in D_3$
$\mathcal{K}_3=\{5\}$

$\mathcal{K}=\mathcal{K}_1\cup\mathcal{K}_2\cup\mathcal{K}_3$
$\mathcal{K}=\{\frac{5}{3}\}\cup\emptyset\cup\{5\}$
$\mathcal{K}=\{\frac{5}{3},5\}$

$b)\ |x-5|-|x-1|=4$    $O=R$, $D=R$

[ nul. b. $x-5=0$    $x-1=0$
$x_0=5$    $x_0=1$ ]



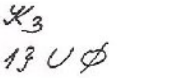
| | $D_1$ $(-\infty,1)$ | $D_2$ $\langle 1,5)$ | $D_3$ $\langle 5,+\infty)$ | |
|---|---|---|---|---|
| $\|x-5\|$ | ⊖ $5-x$ | ⊖ $5-x$ | 0 ⊕ $x-5$ | nr. 6: $6-5>0$ |
| $\|x-1\|$ | ⊖ $1-x$ | 0 ⊕ $x-1$ | ⊕ $x-1$ | $6-1>0$ |

1\. $x\in(-\infty,1)\ \ldots\ 5-x-(1-x)=4$
$D_1=(-\infty,1)$
$5-x-1+x=4$
$0x+4=4$
$0x=0$ pl.
$\mathcal{K}_1=D_1=(-\infty,1)$

2\. $x\in\langle 1,5)\ \ldots\ 5-x-(x-1)=4$
$D_2=\langle 1,5)$
$5-x-x+1=4$
$-2x+6=4$
$-2x=-2$
$x=1\in D_2$
$\mathcal{K}_2=\{1\}$

3\. $x\in\langle 5,+\infty)\ \ldots\ x-5-(x-1)=4$
$D_3=\langle 5,+\infty)$
$x-5-x+1=4$
$0x-4=4$
$0x=8$ nepl.
$\mathcal{K}_3=\emptyset$

$\mathcal{K}=\mathcal{K}_1\cup\mathcal{K}_2\cup\mathcal{K}_3$
$\mathcal{K}=(-\infty,1)\cup\{1\}\cup\emptyset$
$\mathcal{K}=(-\infty,1\rangle$



POZOR!
– opačný výraz k výrazu $x+3$ je $-(x+3)=-x-3$ !
$x-3$ je $-(x-3)=-x+3=3-x$
$3-x$ je $-(3-x)=-3+x=x-3$
$-3-x$ je $-(-3-x)=3+x$

## 7) Řešte v $R$

**NEROVNICE S ABS. HODNOTAMI**

### a) $|u+3| \geq 8 - |2-3u|$  $O = R$, $D = R$

nul. b. $u+3=0$, $u_0=-3$; $2-3u=0$, $2=3u \;/:3$, $u_0=\frac{2}{3}$

| | $D_1$ $(-\infty,-3)$ | $D_2$ $\langle -3,\frac{2}{3})$ | $D_3$ $\langle \frac{2}{3},+\infty)$ | |
|---|---|---|---|---|
| $\|u+3\|$ | ⊖ $-u-3$ | ⊕ $u+3$ | ⊕ $u+3$ | nr. 1: $1+3>0$ |
| $\|2-3u\|$ | ⊕ $2-3u$ | ⊕ $2-3u$ | ⊖ $3u-2$ | $2-3<0$ |

1. $D_1=(-\infty,-3)$ … $-u-3 \geq 8-(2-3u)$

$x \in (-\infty,-3)$ $\quad -u-3 \geq 6+3u$

$-9 \geq 4u \;/:4$

průnik $\quad u \leq -\frac{9}{4}$

$K_1 = (-\infty,-3)$

2. $D_2=\langle -3,\frac{2}{3})$ … $u+3 \geq 8-(2-3u)$

$x \in \langle -3,\frac{2}{3})$ $\quad u+3 \geq 6+3u$

$-3 \geq 2u$

průnik $\quad u \leq -\frac{3}{2}$

$K_2 = \langle -3,-\frac{3}{2}\rangle$

KDYBY
$u+3 \geq 6+3u$
$-2u \geq 3 \;/:(-2)$
$u \overset{!}{\leq} -\frac{3}{2}$
změna zn.

3. $D_3=\langle \frac{2}{3},+\infty)$ … $u+3 \geq 8-(3u-2)$

$x \in \langle \frac{2}{3},+\infty)$ $\;0,\overline{6}$ $\quad u+3 \geq 10-3u$

$4u \geq 7$

$u \geq \frac{7}{4}$ (1,75)

$K_3 = \langle \frac{7}{4},+\infty)$

$K = K_1 \cup K_2 \cup K_3$

$K = (-\infty,-3) \cup \langle -3,-\frac{3}{2}\rangle \cup \langle \frac{7}{4},+\infty)$

$K = (-\infty,-\frac{3}{2}\rangle \cup \langle \frac{7}{4},+\infty)$

### b) $|2x-1| + |x-2| \geq 1$  $O = R$, $D = R$

nul. b. $2x-1=0$, $2x=1$, $x_0=\frac{1}{2}$; $x-2=0$, $x_0=2$

| | $D_1$ $(-\infty,\frac{1}{2})$ | $D_2$ $\langle \frac{1}{2},2)$ | $D_3$ $\langle 2,+\infty)$ | |
|---|---|---|---|---|
| $\|2x-1\|$ | ⊖ $1-2x$ | ⊕ $2x-1$ | ⊕ $2x-1$ | nr. 3: $2\cdot 3-1>0$ |
| $\|x-2\|$ | ⊖ $2-x$ | ⊖ $2-x$ | ⊕ $x-2$ | $3-2>0$ |

1. $D_1=(-\infty,\frac{1}{2})$ … $1-2x+2-x \geq 1$

$[x \in (-\infty,\frac{1}{2})]$ $\quad 3-3x \geq 1$

$-3x \geq -2 \;/\cdot(-1)$ ZMĚNA ZN. NEROV.

průnik $\quad 3x \leq 2$

$x \leq \frac{2}{3}$

$K_1 = (-\infty,\frac{1}{2})$

2. $D_2=\langle \frac{1}{2},2)$ … $2x-1+2-x \geq 1$

$[x \in \langle \frac{1}{2},2)]$ $\quad x+1 \geq 1$

průnik $\quad x \geq 0$

$K_2 = \langle \frac{1}{2},2)$

3. $D_3=\langle 2,+\infty)$ … $2x-1+x-2 \geq 1$

$[x \in \langle 2,+\infty)]$ $\quad 3x-3 \geq 1$

$3x \geq 4$

průnik $\quad x \geq \frac{4}{5}$

$K_3 = \langle 2,+\infty)$

$K = K_1 \cup K_2 \cup K_3$

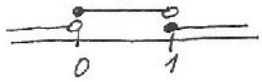
$K = (-\infty,\frac{1}{2}) \cup \langle \frac{1}{2},2) \cup \langle 2,+\infty)$

$K = (-\infty,+\infty) = R$

DÚ c) $|x-1|+|x|>4$

$\sigma = R,\ \mathcal{D} = R$

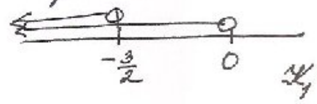
[ nul. b. $x-1=0$ $x_0=1$, $x_0=0$



]

| | $\mathcal{D}_1$ $(-\infty,0)$ | $\mathcal{D}_2$ $\langle 0,1)$ | $\mathcal{D}_3$ $\langle 1,+\infty)$ | |
|---|---|---|---|---|
| $\lvert x-1\rvert$ | $\ominus$ $1-x$ | $\ominus$ $1-x$ | $0$ $\oplus$ $x-1$ | nr. 2: $2-1>0$ |
| $\lvert x\rvert$ | $\ominus$ $-x$ | $0$ $\oplus$ $x$ | $\oplus$ $x$ | nr. 2: $2>0$ |

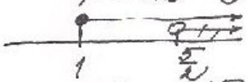
1. $x \in (-\infty,0)$ ... $1-x-x>4$
$[\mathcal{D}_1 = (-\infty,0)]$
$-2x>3$ !
$x<-\frac{3}{2}$
průnik




$\underset{\sim\sim\sim}{\mathcal{K}_1 = (-\infty,-\frac{3}{2})}$

2. $x \in \langle 0,1)$ ... $1-x+x>4$
$[\mathcal{D}_2 = \langle 0,1)]$
$0x>3$
nepl.
$\underset{\sim\sim\sim}{\mathcal{K}_2 = \emptyset}$

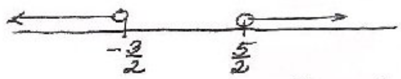
3. $x \in \langle 1,+\infty)$ ... $x-1+x>4$
$[\mathcal{D}_3 = \langle 1,+\infty)]$
$2x-1>4$
$2x>5$
průnik
$x>\frac{5}{2}$




$\underset{\sim\sim\sim}{\mathcal{K}_3 = (\frac{5}{2},+\infty)}$

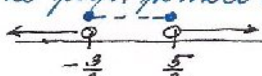
$\mathcal{K} = \mathcal{K}_1 \cup \mathcal{K}_2 \cup \mathcal{K}_3$

$\mathcal{K} = (-\infty,-\frac{3}{2}) \cup \emptyset\ (\frac{5}{2},+\infty)$




$\underset{\sim\sim\sim}{\mathcal{K} = (-\infty,-\frac{3}{2}) \cup (\frac{5}{2},+\infty)}$

[ lze přep. pomocí doplňku do R



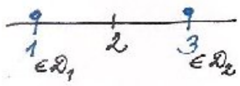

$\mathcal{K} = \langle -\frac{3}{2},\frac{5}{2}\rangle'_R$ ]

⑧ Řešte v R    TYP VNOŘENÉ ABS. HODNOTY

a) $||x+1|-3|=1$    $\sigma = R,\ \mathcal{D} = R$

Z DEF. ABS. H.

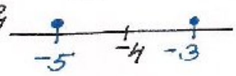
1. zp. 1. $x+1 \geq 0$ ... $|x+1-3|=1$
$x \geq -1$
$\mathcal{D}_1 = \langle -1,+\infty)$
$|x-2|=1$
$[x_0=2]$




$\underset{\sim\sim\sim}{\mathcal{K}_1 = \{1,3\}}$

2. $x+1<0$ ... $|-x-1-3|=1$
$x<-1$
$\mathcal{D}_2 = (-\infty,-1)$
$|-x-4|=1$
$|(-1)(x+4)|=1$
$|x+4|=1$
$[x_0=-4]$

$\mathcal{K}_2 = \{-5,-3\}$



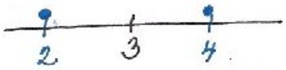

$\underset{\sim\sim\sim}{\mathcal{K} = \mathcal{K}_1 \cup \mathcal{K}_2 = \{1,3,-3,-5\}}$

2. zp. SUBSTITUCE (další metoda bude probírána později)

$||x+1|-3|=1$    $\sigma = R$  $\mathcal{D} = R$

$\underset{\sim\sim\sim}{\text{subst. } y = |x+1|}$

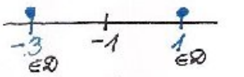
$|y-3|=1$
$[y_0=3]$




$y_1 = 2$    $y_2 = 4$

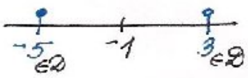
ZPĚT K SUBSTITUCI ($y = |x+1|$)

$y_1 = 2$
$|x+1| = 2$
$[x_0=-1]$




$\mathcal{K}_1 = \{-3,1\}$

$y_2 = 4$
$|x+1| = 4$
$[x_0=-1]$




$\mathcal{K}_2 = \{-5,3\}$

$\underset{\sim\sim\sim}{\mathcal{K} = \mathcal{K}_1 \cup \mathcal{K}_2 = \{1,3,-3,-5\}}$

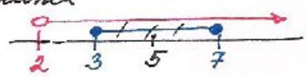
b) $|3-|2-x||\leqq 2$ $\quad O=R,\ D=R$

1.způs. Z DEF. ABS. H.

1. $2-x\geqq 0 \ \dots\ |3-(2-x)|\leqq 2$
$-x\geqq -2 \qquad |1+x|\leqq 2$
$x\leqq 2 \qquad |x+1|\leqq 2$
$D_1=(-\infty,2\rangle$ průnik $[x_0=-1]$ na základě geom. významu abs. h.


$\underline{K_1=\langle -3,1\rangle}$

2. $2-x<0 \ \dots\ |3-(x-2)|\leqq 2$
$-x<-2 \qquad |5-x|\leqq 2$
$x>2 \qquad |a-b|=|b-a|$
$D_2=(2,+\infty) \qquad |x-5|\leqq 2$
$[x_0=5]$
průnik



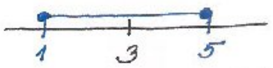

$\underline{K_2=\langle 3,7\rangle}$

sjednocení

$$\underline{K=K_1\cup K_2=\langle -3,1\rangle\cup\langle 3,7\rangle}$$

2.způs. SUBSTITUCE

$|3-|2-x||\leqq 2$
subst. $y=|2-x|$
$|3-y|\leqq 2$
$|y-3|\leqq 2$
$[y_0=3]$

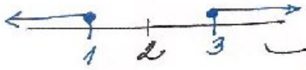


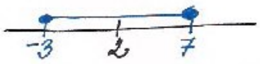

$y\in\langle 1,5\rangle$, tj.:
! $1\leqq y\leqq 5$

$1\leqq y\leqq 5$
zpět k subst.
$1\leqq|2-x|\leqq 5$

$1\leqq|2-x|$ a současně $|2-x|\leqq 5$

$|2-x|\geqq 1 \qquad |x-2|\leqq 5$
$|x-2|\geqq 1 \qquad [$nul.b. $x_0=2]$
$[$nul.b. $x_0=2]$



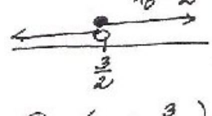

průnik


$\underline{K=\langle -3,1\rangle\cup\langle 3,7\rangle}$

9) ŘEŠTE v R, POROVNEJTE S JINÝMI TYPY ŘEŠENÍ (viz př. 4a), 5b))
METODOU NUL. BODŮ

a) $|2x-3|=1$

$[$nul.b. $2x-3=0$
$2x=3$
$x_0=\frac{3}{2}]$




| | $D_1$ $(-\infty,\frac{3}{2})$ | $\langle\frac{3}{2},+\infty)$ |
|---|---|---|
| $\lvert 2x-3\rvert$ | ⊖ $3-2x$ | ⊕ $2x-3$ |

zk. 2: $2\cdot 2-3>0$

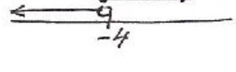
1. $D_1=(-\infty,\frac{3}{2})\ \dots\ 3-2x=1$
$-2x=-2$
$x=1\in D_1$
$K_1=\{1\}$

2. $D_2=\langle\frac{3}{2},+\infty)\ \dots\ 2x-3=1$
$2x=4$
$x=2\in D_2$
$K_2=\{2\}$

$K=K_1\cup K_2$
$\underline{K=\{1,2\}}$

b) $|x+4|-2\geqq 4(x-3)$

$[$nul.b. $x+4=0$
$x_0=-4]$




| | $D_1$ $(-\infty,-4)$ | $\langle -4,+\infty)$ |
|---|---|---|
| $\lvert x+4\rvert$ | ⊖ $-x-4$ | ⊕ $x+4$ |

zk. 0: $0+4>0$

DÁLE ŘEŠÍME viz 5b)
1. $x\in(-\infty,-4)\ \dots\ -x-4-2\geqq 4(x-3)$
2. $x\in\langle -4,+\infty)\ \dots\ x+4-2\geqq 4(x-3)$